Power flow optimization in the presence of microgrids

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1 Introduction

Power flow optimization problems consist in generating electricity from different types of power plants in order to attend the demand of the electrical network. Many different types of such problems exist, depending on the type of electrical network (transmission, distribution, ...) and the types of power plants considered (thermic, nuclear, solar, wind, hydro, ...). Even when the context is well specified (type of network and power plants), the true resulting mathematical optimization problems are often intractable due the presence of discrete decisions variables and non-convexities in the functions describing the technical constraints. As a result, one should also decide of the level of simplification used to model the electrical power flows (AC model, DC model, or bus model) and the technical constraints faced by the power plants. We can mention, among other, the following three types of power flow optimization problems that have heavily been studied in the literature, each of them resulting in a large scale multi-stage stochastic mixed-integer non-linear program:

- Unit-commitment (e.g., [3]): how to schedule the production of plants (thermal, hydro) today for tomorrow in the most cost-effective way.
- Maintenance scheduling of large power plants (e.g., cf. ROADEF Challenge 2010): when to shut down nuclear power plant to perform maintenance.
- Hydro resource scheduling: compute optimal use of water over a pluri-annual (say bi-annual) time horizon while accounting for uncertainty on inflows. Water has no cost, it is obtained as a substitution cost.

Each of these problems is already very difficult to solve to optimality with real data sets. On the top of this already complex picture, one should also consider the generation uncertainty of the wind and solar plants pertaining to the network as well as game-theoretical aspects to model the prices offered by the different electricity producer on the market.

Yet, networks of future generation will introduce even more complexity to the situation by replacing the traditional and centralized power transmission networks with smart-grids:

> *"A smart grid is an electrical grid which includes a variety of operational and energy measures including smart meters, smart appliances, renewable energy resources, and energy efficiency resources"* (Wiki definition).

Figure 1 shows an overview of a smart grid system.

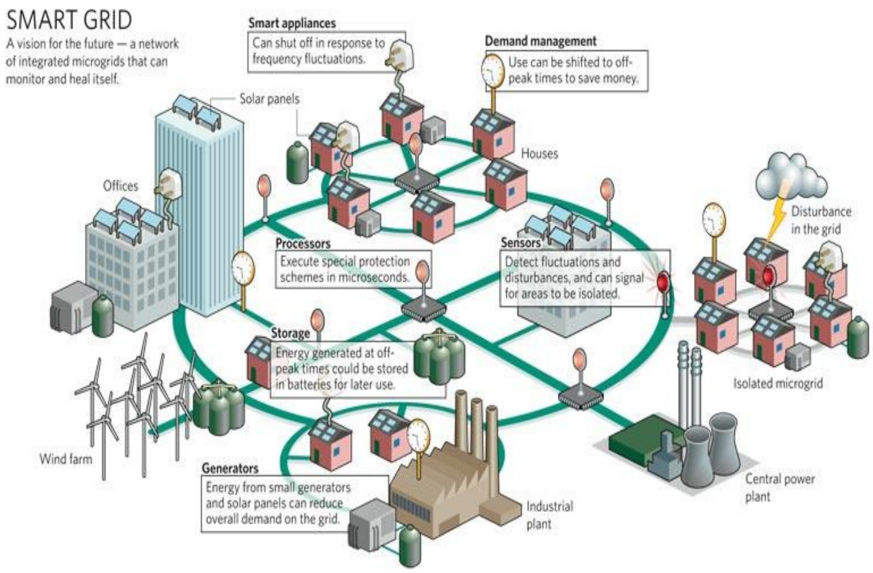

Figure 1: An overview of a smart grid system.

The essence of the smart grid is an advanced energy management system interacting with programmable elements in the grid including good monitoring and control functions, a pervasive communication system and specific items such as smart meters, programmable loads, switchable storage systems and a variety of controllable energy sources including solar, wind and wave generators. The smart grid is sustainable, reliable, economic, and tuned to optimise the benefits to members of the smart microgrid and to interact predictably with any major grids interconnected with it. They will increase energy efficiency and smoothly integrate low carbon energy sources. Some of the major changes introduced by smart grids are the following:

- Micro-grids: smaller nearly isolated sub-grids that interact only with the global system when a load/offer mismatch occurs. Most importantly these sub-grids can be managed to follow a local economical target (which may be different and contrary to a system-wide interest)
- Partial storage, perhaps through electrical vehicles
- Demand management tools: use advanced IT to pilot electricity use: for instance shut down electrical heating, reprogram hot water tank recharging etc...

The importance of these topics to the international community is such that a very large consortium has recently been awarded funds from the european comission to work on these issues (http://smartnet-project.eu/).

Our approach In this document, we focus on the relation between the main grid and the microgrids. We assume that the GenCo proposes contracts to the microgrids that detail the price of buying/selling electricity to the network. This naturally leads to bilevel multi-stage mixed-integer stochastic program. Yet, we show in the manuscript that the problem has a one-level reformulation, that does not seem much more complex than solving the aforementioned power flow optimization problems. The assumption of our model are further described in Section 2. Section 3 then proposes the mathematical formulation for the problem. Finally, Section 4 proposes a detailed flow formulation for the optimization problem faced by each microgrid.

2 The optimization problem

We consider an extension of electricity production problems that involve two types of parties:

- **GenCos** are big producers of electricity in the network (nuclear, thermal, hydro and other renewable energies). To avoid game-theory aspects, we focus on the decision of a single operator that faces concurrent companies that have a fixed, perfectly known policy.
- Each **microgrid** $q \in Q$ consists of a small subnetwork that has highly volatile generation capacities (solar, wind), and two types of demands. On the one hand, there is a hard demand that must be attended at a given time period, while on the other hand, there is an elastic demand (heating up water, recharging electric car, ...) that must be attended during a group of time periods. Micrograds are assumed to be relatively autonomous in terms of energy. However, due to the uncertain nature of their production, they need to buy or sell electricity from the GenCo.

Contract Micrograds and GenCos interact through contracts that specify the costs of buying/selling electricity from/to the GenCo for each period of the time horizon T. Specifically, each contract $k \in K$ is specified by (i) the price of contract c_k paid by the microgrid to the GenCo that proposes it, (ii) a linear cost function $f_{kt}x$ for buying the amount x of electricity during time period t, and (iii) a linear function $g_{kt}y$ for selling the amount y of electricity during time period t. We denote by K_0 the subset of contracts possibly proposed by the GenCo whose decisions are being optimized, while K also contains contracts of concurrent companies. We assume that f is linear for tractability issue as real data suggest that the function is slightly concave. Notice that we need functions for buying and selling electricity because the uncertain nature of the problem implies that we do not know in advance how much electricity shall be bought/sold by the the microgrids.

Objective On the one hand, the objective of the GenCo is to propose the least cost production schedule based on (i) the classical costs of unit commitment and related problems (hydro optimization and nuclear outage scheduling) and (ii) the cost/benefit of buying/selling electricity to the microgrids. On the other hand, the objective of each microgrid is to minimize its total cost of buying/selling electricity to the GenCos, by choosing a contract offered by the GenCo or one of the concurrent companies (in the latter case, the GenCo does not produce, buy or earn anything for/from the microgrid). These two conflicting objectives can be naturally modeled as a bilevel optimization problem.

Temporal and nature of the problem - uncertainty We are given a time horizon $\mathcal{T} = \{1, \dots, T\}$ that is further partitioned into days: $\mathcal{T} = \mathcal{D}_1 \cup \dots \cup \mathcal{D}_J$. At the first time period of each day $j \in \{1, \dots, J\}$ we know the weather for the entire day $\mathcal{D}_j$, and therefore, the partial scenario containing the informations for all time periods up to that day (denoted $\xi_{j(t)}$). Then, the problem is a multistage stochastic program with the following decisions:

- *First stage*
  - The GenCo chooses a set of contracts that are compatible with its objective. This can be done by introducing binary optimization variables.
  - Each microgrid chooses a contract among the contracts offered by the GenCos.
- *Subsequent stages* The exact productions and demands are known for all entities for all time periods that belong to the current day. Hence, the GenCo can produce the electricity and the microgrids manage their elastic loads and batteries according to the chosen contracts and generated power. For simplicity we consider the bus-model for the microgrids.

Coupling among the different time periods Elastic loads couple all time periods of a given day. However, they do not couple time periods of different days as we may reasonably assume that the required load will be attended during one full day. Nevertheless, time periods of different days may be coupled together in the presence of batteries. This hardens substantially the multi-stage stochastic optimization problem, preventing us from decomposing the problem by day.

3 The mathematical formulation

We describe below our mathematical model, represented as a bilevel multi-stage stochastic program.

3.1 Deterministic bilevel problem

Let $x \in \mathbb{R}^{T \times Q}$ represent the electricity production of the GenCo. We are mainly interested here at the interaction between the GenCo and the microgrids, so the value x_{qt} represents the electricity produced by the GenCo and fed into microgrid q during period t. Similarly, we can define y_{qt} as the amount of electricity bought by the GenCo from microgrid q during time period t. We denote by $F : \mathbb{R}^{2T\times Q} \to \mathbb{R}$ the cost of producing $x - y$. Hence, one can think of the problem

$$\min_{x,y\geq 0} F(x, y)$$

as a compact representation for the combination of unit commitment, nuclear power plant maintenance planning, and hydro power generation, and other related problems. In a sense, we hide the difficulty of computing accurate power flows and their cost inside the minimization of function F.

In our bilevel problem, x and y are decision variables fixed by the microgrids since they only indicate the power flows linked directly to the microgrids. These variables should satisfy the constraints related to the functioning of the microgrids, represented by $\mathcal{M}_q$ for each $q \in Q$. In contrast, the GenCo has to decide of the contracts it proposes to the microgrids. Hence, we introduce the additional binary variables Z_{qk} that is equal to 1 if contract k is offered to microgrid q. We also impose that the GenCo must propose a fixed number of contracts N_q to each microgrid. Then, for each microgrid $q \in Q$, binary variable z_{qk} indicates if microgrid q subscribes to contract k.

Summarizing, the deterministic bilevel problem below considers the following optimization variables:

- Z_{qk}: 1 iif contract k is offered to microgrid q (*leader*)
- z_{qk}: 1 iif contract k is subscribed by microgrid q (*follower*)
- x_{qt}: power consumed by micro-grid q during period t (*follower*)
- y_{qt}: power produced by micro-grid q during period t (*follower*)

$$\begin{aligned}
\min \quad & F(x,y) - \sum_{q\in Q}\sum_{k\in K_0}\sum_{t\in\mathcal{T}} \left(f_{kt}x_{qt} - g_{kt}y_{qt} + c_k\right) z_{qk} \\
\text{s.t.} \quad & \sum_{k\in K_0} Z_{qk} = N_q, \quad \forall q \in Q \\
& Z \in \{0,1\}^{|K_0|\times|Q|} \\
& (x_q, y_q, z_q) \in \arg\min \sum_{k\in K}\sum_{t\in\mathcal{T}} \left(f_{kt}x_{qt} - g_{kt}y_{qt} + c_k\right) z_{qk}, \quad \forall q \in Q \\
& \qquad\qquad \text{s.t.} \;\; (x_q, y_q) \in \mathcal{M}_q, \quad \forall q \in Q \\
& \qquad\qquad\qquad z_{qk} \leq Z_{qk}, \quad \forall q \in Q, k \in K_0 \\
& \qquad\qquad\qquad \sum_{k\in K} z_{qk} = 1, \quad \forall q \in Q \\
& \qquad\qquad\qquad z \in \{0,1\}^{K\times Q} \\
& \qquad\qquad\qquad x, y \geq 0
\end{aligned}$$

When multiple follower solutions are available, bilevel optimization usually assumes that the follower takes the solution that mostly benefits the leader, often called the *optimistic* assumption. We propose below a one-stage reformulation of the bilevel problem that may not validate the assumption. The key aspect of our reformulation relies on pre-processing. Specifically, for each $k \in K$ and $q \in Q$, we solve the restricted follower problem where z_{qk} is equal to 1, namely:

$$
\begin{aligned}
\min \quad & \sum_{t \in \mathcal{T}} f_{kt} x_{qt} - g_{kt} y_{qt} + c_k, \quad \forall q \in Q \\
\text{s.t.} \quad & (x_q, y_q) \in \mathcal{M}_q, \quad \forall q \in Q \\
& x, y \geq 0.
\end{aligned}
$$

Let $(\overline{x}_{qk}, \overline{y}_{qk})$ be an optimal solution of the above problem and $\overline{C}_{qk}$ be its solution cost. Then, the above bilevel problem is equivalent to

$$
\begin{aligned}
\min \quad & F(x,y) - \sum_{q \in Q} \sum_{k \in K_0} \overline{C}_{qk} z_{qk} \\
\text{s.t.} \quad & \sum_{k \in K_0} Z_{qk} = N_q, \quad \forall q \in Q \\
& z_{qk} \leq Z_{qk}, \quad \forall q \in Q, k \in K_0 \\
& \sum_{k \in K} z_{qk} = 1, \quad \forall q \in Q \\
& x_{qt} = \sum_{k \in K} \overline{x}_{qkt} z_{qk}, \quad \forall q \in Q, \forall t \in \mathcal{T} \\
& y_{qt} = \sum_{k \in K} \overline{y}_{qkt} z_{qk}, \quad \forall q \in Q, \forall t \in \mathcal{T} \\
& z_{qk} \leq 1 - Z_{\ell q}, \quad \forall k \in K, l \in K, q : \overline{C}_{qk} > \overline{C}_{\ell q} \\
& Z \in \{0,1\}^{K \times Q} \\
& z \in \{0,1\}^{K \times Q}
\end{aligned}
$$

Our reformulation may not yield a solution compatible with the optimistic assumption because whenever multiple optimal solutions exist to the subproblems, we chose $(\overline{x}, \overline{y})$ arbitrarily.

3.2 Stochastic bilevel problem

Power flow problems that involve renewable energy like wind and solar are subject to uncertain power generation, since the output of the renewable power plants depend on the weather condition. In the best case, the latter is known a few hours in advance. We denote by Ξ the set of scenarios and redefine x and y as $x(\xi)$ and $y(\xi)$, since their values now depend on the scenario under consideration. To keep concise notations, we do not express the non-anticipativity constraints explicitly. Instead, we implicitly assume that $x_t(\xi)$ and $y_t(\xi)$ depend on the partial scenario $\xi_{j(t)}$ that contains only the informations that have been revealed at time period t. More specifically, $\xi_{j(t)}$ contains the weather conditions on all days in $\{1, \ldots, j\}$. In contrast, variables z and Z are first-stage variables that represent decisions taken at the very beginning of the decision process. Hence, they are independent of ξ.

The introduction of stochasticity affects both the objective function and the constraints of the follower problem, as well as the objective function of the leader. Specifically, we denote by $\mathcal{M}_q(\xi)$ the constraints that $(x(\xi), y(\xi))$ must satisfy. The objective functions now involve two terms: one the one hand, one wishes to optimize the average value, represented by the expectation $\mathbb{E}$, while on the other hand we would like to avoid extreme values, which can be represented by a risk measure such as CVaR_ϵ, defined as

$$\mathrm{CVaR}_\epsilon[X(\xi)] = \min_v \left\{ v + \frac{1}{\epsilon}\mathbb{E}[(X(\xi)-v)^+] \right\}$$

for any random variable X and probability level $\epsilon \in (0,1)$. As often in stochastic programming, it is left to the decision maker in charge to choose which of the two terms is more important, yielding a bi-objective optimization problem. Herein, we simply assume that each decision maker decides of a pair of weights $(\lambda, 1-\lambda)$ that she wishes to affect to both terms, where $\lambda \in [0,1]$. We let λ^0 be related to the GenCo while the weights of the microgrids are denoted by λ^q for each $q \in Q$. To shorten notations, we denote the objective function of microgrid q as

$$F^q(x_q(\xi), y_q(\xi), z_q) := \sum_{k\in K}\sum_{t\in\mathcal{T}} \left(f_{kt}x_{qt} - g_{kt}y_{qt} + c_k\right) z_{qk},$$

and denote the associated payoff for the GenCo as:

$$F_0^q(x_q(\xi), y_q(\xi), z_q) := \sum_{k\in K_0}\sum_{t\in\mathcal{T}} \left(f_{kt}x_{qt} - g_{kt}y_{qt} + c_k\right) z_{qk},$$

obtaining the following bilevel stochastic optimization problem.

$$\begin{aligned}
\min \quad & \lambda^0\mathbb{E}\left[F(x(\xi), y(\xi)) - \sum_{q\in Q} F_0^q(x_q(\xi), y_q(\xi), z_q)\right] \\
& + (1-\lambda^0)\mathrm{CVaR}_\epsilon\left[F(x(\xi), y(\xi)) - \sum_{q\in Q} F_0^q(x_q(\xi), y_q(\xi), z_q)\right] \\
\text{s.t.} \quad & \sum_{k\in K} Z_{qk} = N_q, \quad \forall q \in Q \\
& Z \in \{0,1\}^{K\times Q} \\
& x_0(\xi) = \sum_{q\in Q} x_q(\xi) \sum_{k\in K_0} z_{kq}, \quad y_0(\xi) = \sum_{q\in Q} y_q(\xi) \sum_{k\in K_0} y_{kq} \\
& (x_q, y_q, z_q) \in \arg\min \ \lambda^q\mathbb{E}\left[F^q(x_q(\xi), y_q(\xi), z_q)\right] \\
& \qquad\qquad + (1-\lambda^q)\mathrm{CVaR}_\epsilon\left[F^q(x_q(\xi), y_q(\xi), z_q)\right], \ \forall q \in Q \\
& \qquad \text{s.t.} \ (x_q(\xi), y_q(\xi)) \in \mathcal{M}_q(\xi), \quad \forall q \in Q, \xi \in \Xi \\
& \qquad\qquad z_{qk} \le Z_{qk}, \quad \forall q \in Q, k \in K_0 \\
& \qquad\qquad \sum_{k\in K} z_{qk} = 1, \quad \forall q \in Q \\
& \qquad\qquad z \in \{0,1\}^{K\times Q} \\
& \qquad\qquad x, y \ge 0
\end{aligned}$$

As in the deterministic case, the problem can be greatly simplified by solving follower problems in a pre-processing phase. For each $k \in K$ and $q \in Q$, we solve the restricted follower problem where z_{qk} is equal to 1

$$
\begin{aligned}
\min \quad & \lambda^q \mathbb{E}\left[F^q(x_q(\xi), y_q(\xi), z_q)\right] + (1-\lambda^q)\mathrm{CVaR}_\epsilon\left[F^q(x_q(\xi), y_q(\xi), z_q)\right], \quad \forall q \in Q \\
\text{s.t.} \quad & (x_q(\xi), y_q(\xi)) \in \mathcal{M}_q(\xi), \quad \forall q \in Q, \xi \in \Xi \\
& x, y \geq 0,
\end{aligned}
$$

and let $(\overline{x}_{qk}, \overline{y}_{qk})$ be an optimal solution and $\overline{C}_{qk}$ be its solution cost. We obtain the following reformulation

$$
\begin{aligned}
\min \quad & \lambda^0 \mathbb{E}\left[F(x(\xi), y(\xi)) - \sum_{q \in Q} F_0^q(x_q(\xi), y_q(\xi), z_q)\right] \\
& + (1-\lambda^0)\mathrm{CVaR}_\epsilon\left[F(x(\xi), y(\xi)) - \sum_{q \in Q} F_0^q(x_q(\xi), y_q(\xi), z_q)\right] \\
\text{s.t.} \quad & \sum_{k \in K_0} Z_{qk} = N_q, \quad \forall q \in Q \\
& z_{qk} \leq Z_{qk}, \quad \forall q \in Q, k \in K_0 \\
& \sum_{k \in K} z_{qk} = 1, \quad \forall q \in Q \\
& x_{qt}(\xi) = \sum_{k \in K} \overline{x}_{qkt}(\xi) z_{qk}, \quad \forall q \in Q, \forall t \in \mathcal{T}, \xi \in \Xi \\
& y_{qt}(\xi) = \sum_{k \in K} \overline{y}_{qkt}(\xi) z_{qk}, \quad \forall q \in Q, \forall t \in \mathcal{T}, \xi \in \Xi \\
& z_{qk} \leq 1 - Z_{\ell q}, \quad \forall k, q : \overline{C}_k > \overline{C}_{\ell q} \\
& Z \in \{0,1\}^{K \times Q} \\
& z \in \{0,1\}^{K \times Q}
\end{aligned}
$$

We point out that, unlike the deterministic case, we cannot substitute the terms of the objective function related to the microgrid costs by

$$
\overline{C}_{qk} = \mathbb{E}\left[\sum_{t \in \mathcal{T}} f_{kt} x_{qt} + g_{kt} y_{qt} + c_k\right] + \mathrm{CVaR}_\epsilon\left[\sum_{t \in \mathcal{T}} f_{kt} x_{qt} + g_{kt} y_{qt} + c_k\right]
$$

because of the non-linearity of CVaR_ϵ.

4 Detailing the microgrid problems

4.1 Context

> *"A microgrid is a discrete energy system consisting of distributed energy sources (including demand management, storage, and generation) and loads capable of operating in parallel with, or independently from, the main power grid."* (Wiki definition)

A microgrid is a low power (compared with the network), low voltage electric power system containing consumers and producers of power and an energy management system. The consumers of power may be fixed, switchable and controllable loads. The producers of power may include renewable generation systems. There may be some storage systems capable of holding significant proportions of the total consumption load [1].

A microgrid may vary in size from a single household to a larger area like a University campus, a township or a small city. The microgrid has a reduced dependence on the main grid and may even go off-grid for periods of time. Power producers in the microgrid may supply consumers in the grid, may export power to the main grid or may direct it to storage devices for extraction when they are not themselves producing power. These two modes of operation of the microgrid allow it to be isolated from the main grid when the main grid experiences a failure or to connect with the main grid at times when demand exceeds microgrid supply.

The energy management system and its communication system have to monitor instantaneous demand and production and to optimize the costs charged for extraction of power from the main grid and prices paid for the supply of power to the main grid. It has to control switchable loads and schedule timed consumption of power. Finally it has to maintain a dialogue with the energy management system of the main grid to ensure that both are fully aware of the options available to them.

Figure 2 shows a microgrid system with a wind farm, electrical and thermal energy storage, hotel, hospital, and residences (see [2]).

For simplicity, this specific study considers the bus model for power flows, that is, the power network is not taken into consideration. We define two types of unit components in the microgrid, which are called devices in the sequel. *Storage* devices typically represent batteries, whose status may switch between online (connected to the grid) and offline during the time horizon. During its offline periods, a storage device can be unloaded: for example, a battery car is loaded during the night. During this time period, it can be used to store and serve power, but it must be fully loaded at the end of the night. During the day, the car is used and its battery is emptied so that its storage level is low when it is back online. In our setting, we associate with each storage device a set of online time intervals. At the beginning of such a time interval, the charge level is a stochastic input parameter (for example, it depends on how much the car has been used during the day). The required charge level at the end of the online period is modeled through more general parameters giving minimum and maximum acceptable charge levels for each online time step. We also assume that each storage device has limited capacity, charging and discharging speed and a power loss factor that is the proportion of power stored to the power consumed during the charge.

The *regular* devices come with stochastic consumptions and productions of power during each time step. Some of their consumption can be partially delayed (elastic demand). We model this feature by defining a set of time intervals for each device (for example, a water heater must heat the water during the night). During each of them, the required total power consumption is known, as a stochastic input data. The maximum power consumption of devices is limited during each time step.

The decisions to be taken in the microgrid problem are, first, to choose a contract among those proposed by the GenCos. Then, for each regular device and each time step, the amount of elastic power consumption must be deter-

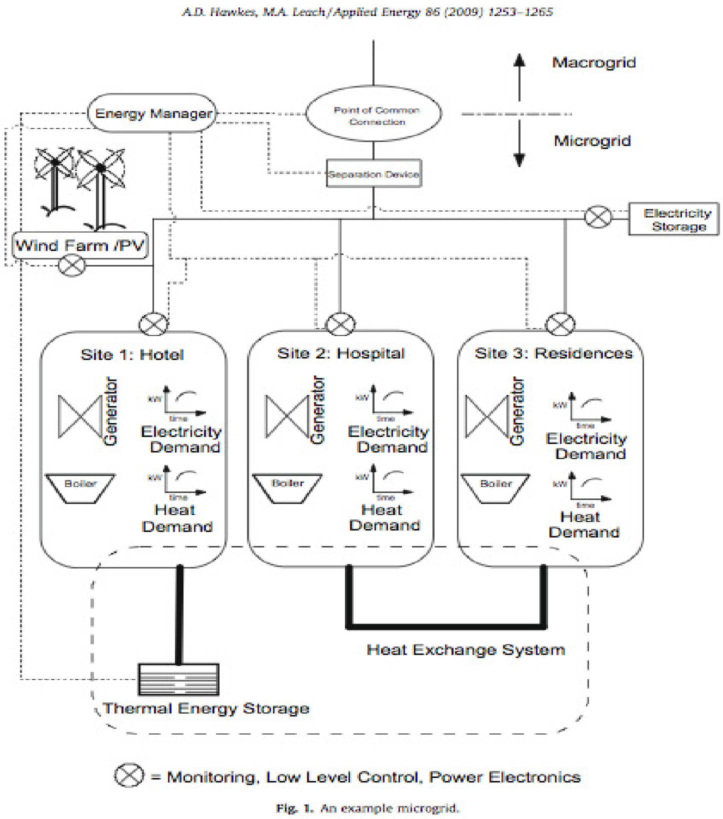

Figure 2: A microgrid system with a wind farm, electrical and thermal energy storage, hotel, hospital, and residences [2].

mined. For each *storage* device, the amounts of power consumed (to charge) and released must be fixed.

4.2 Mathematical formulation

We provide below a model for the problems faced by each one of the microgrids.

Parameters
Scenario-independent parameters:

- K: set of available contracts, defined by c_k, f_k and g_k. We assume that $\forall t \in \mathcal{T}$, $f_{kt} \geq g_{kt}$.
- D: set of non-storage devices. For each device $d \in D$:
  - Θ_D^d: set containing sets of time periods defining elastic consumption slots
- S: set of storage devices. For all $d \in S$:
  - $\bar{s}^d$: capacity of storage device d
  - $\bar{\ell}^d$: maximum power used to reload d during one time period
  - $\bar{u}^d$: maximum power released by d during one time period
  - α^d: power loss factor when charging d
  - Θ_S^d: set of time intervals when d is online (can be charged or discharged). We note, for all $\theta \in \Theta_S^d$, $\theta = [t^-(\theta), t^+(\theta)]$.
  - $\underline{S}_t^d, \bar{S}_t^d$: minimum and maximum charge level for d at time t.

Scenario-dependent parameters: For all scenario $\xi \in \Xi$

- For each device $d \in D$:
  - $\forall t \in T$:
    - $b_t^d(\xi)$: power production of d during period t in scenario ξ
    - $r_t^d(\xi)$: power consumption of d during period t in scenario ξ
    - $\bar{w}_t^d(\xi)$: maximum possible elastic consumption of d during t
  - $\forall \theta \in \Theta_D^d \cup \Theta_S^d$
    - $e_\theta^d(\xi)$: total elastic power demand of d during θ in scenario ξ
  - For each storage device $d \in S$ and online interval $\theta \in \Theta_S^d$: $I_\theta^d(\xi)$ is the initial stock level when d is plugged in.

Decision variables

- Stage 0:
  - $\forall k \in K$: $z_k = 1$ if contract k is chosen by the micro-grid, 0 otherwise
- Stage $t, t \in \mathcal{T}$:

- $x_t(\xi)$: power consumed by the micro-grid during period t
- $y_t(\xi)$: power produced by the micro-grid during period t
  * $\forall d \in D$, $w_t^d(\xi)$: elastic power consumed by d during t
  * For all $d \in S$:
    · $s_t^d(\xi)$: power stock in d at the end of t
    · $\ell_t^d(\xi)$: power consumed to charge device d during t
    · $u_t^d(\xi)$: power released by discharging device d during t

Mathematical programming model

$$\min \sum_{t\in\mathcal{T}} f_{kt} x_t(\xi) + g_{kt} y_t(\xi) \tag{1}$$

$$x_t(\xi) - y_t(\xi) = \sum_{d\in D} \left(r_t^d(\xi) + w_t^d(\xi) - b_t^d(\xi)\right) + \sum_{d\in S} \left(\ell_t^d(\xi) - u_t^d(\xi)\right) \qquad \forall t, \xi \tag{2}$$

$$s_t^d(\xi) = s_{t-1}^d(\xi) + \alpha^d \ell_t^d(\xi) - u_t^d(\xi) \qquad \forall d \in S, \theta \in \Theta_S^d, t \in \theta - \{t^-(\theta)\}, \xi \tag{3}$$

$$s_{t^-(\theta)}^d = I_\theta^d(\xi) + \alpha^d \ell_{t^-(\theta)}^d(\xi) - u_{t^-(\theta)}^d(\xi) \qquad \forall d \in S, \theta \in \Theta_S^d, \xi \tag{4}$$

$$\sum_{t\in\theta} w_t^d(\xi) = e_\theta^d(\xi) \qquad \forall d \in D, \theta \in \Theta_D^d, \xi \tag{5}$$

$$w_t^d(\xi) \leq \bar{w}^d \qquad \forall d \in D, t, \xi \tag{6}$$

$$\underline{S}_t^d \leq s_t^d(\xi) \leq \bar{S}_t^d \qquad \forall d \in S, t, \xi \tag{7}$$

$$\ell_t^d \leq \bar{\ell}^d \qquad \forall d \in S, t \tag{8}$$

$$u_t^d \leq \bar{u}^d \qquad \forall d \in S, t \tag{9}$$

$$\ell_t^d(\xi) = u_t^d(\xi) = 0 \qquad \forall d \in S, \theta \notin \Theta_S^d, t \in \theta \tag{10}$$

$$y_k \in \{0, 1\} \qquad \forall k \in K \tag{11}$$

$$x, y, w, r, s, l, u \geq 0 \tag{12}$$

The objective of the problem (1) is to minimize the total cost for the micro-grid, which is composed of the fixed cost of the contract, the cost of buying power from the GenCos minus the income obtained from selling the over-production. Constraints (2) ensure that the power flow into/out of the microgrid is equal to its production/consumption during each time step. In the right-hand-side, the summation over D (resp. S) represents the total consumption/production of regular (resp. storage) devices. Constraints (3) and (4) define the level of power stock for each device and time step. Constraints (5) fix the correct total amount of power that must be consumed by a device during an elastic consumption interval. The instantaneous power consumed by a device is limited by Constraints (6). The acceptable stock levels are bound by Constraints (7). Constraints (8) and (9) define maximum charging and discharging speeds for the storage devices, while Constraints (10) are just a way to state that an offline device cannot be charged or discharged (the corresponding variables may as well be omitted in the model). The domains of the variables are given in Constraints (11) and (12).

4.3 Minimum cost flow representation

Let us assume that the contract is fixed and data are deterministic. The problem can then be represented as a minimum cost flow problem with gains and loss, which can be solved with dedicated algorithms (see e.g. [4]) or as linear programs. The core structure of the network is described in Figure 3. It is composed of a source node S, that delivers power, and a sink node T that receives all the power that transits in the network. A set of time step-nodes $(t_q)_{q\in\mathcal{T}}$ represents the set of time periods (equivalent to Constraints (2) in the model). For each regular device d and elastic consumption interval θ, we define a node (d, θ) (Constraints (5)). The corresponding elastic demand e^d_θ is represented by an arc $((d, \theta), T)$ on which the lower and upper flow bounds are equal to the demand. For each $q \in \mathcal{T}$, two arcs from S to t_q represent respectively the combined total power production of all devices during period q and the power bought from the GenCos during q (aggregated parameters $\sum_d b^d_q$ and variable x_q). The power entering node t_q through this arc is split among the following out-going arcs:

- one arc to the sink T that represents the total fixed power consumption (whose flow is fixed to the combined total fixed power consumption of all devices, aggregated parameters $\sum_d r^d_q$),
- one arc to the sink T that represents the over-production during q that is sold to the GenCos (variable y_q),
- one arc for each elastic demand interval and device to the node (d, θ), that represents the amount of power provided to that device during period q (variable w^d_q).

This core network can be augmented to handle storage devices as shown in Figure 4. With each online time q period of each storage device d, we associate a node (d, q) that represents the balance of power flow for d during q (Constraints (3) and (4)).

- If q is the first period of the online time interval, then an arc from S provides the initial charge of the device (parameter I^d_θ).
- If $q + 1$ is in the same online time interval, an arc to $(d, q + 1)$ represents the amount of storage available at the end of period q that is transferred to the next period (variable s^d_t).
- If q is the last period of the online time interval, then an arc to the sink ensures that the charge level is sufficient (specific Constraints (7)).

One ingoing arc from (resp. to) t_q represents the amount of power used to charge (resp. discharge) device d during period q, with a loss factor α^d.

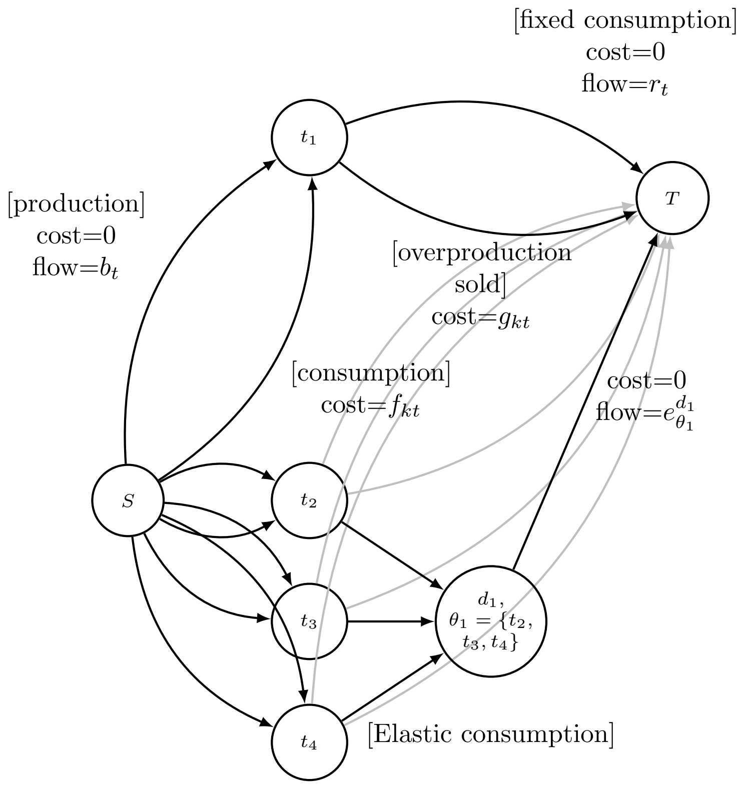

Figure 3: Core structure of the network.

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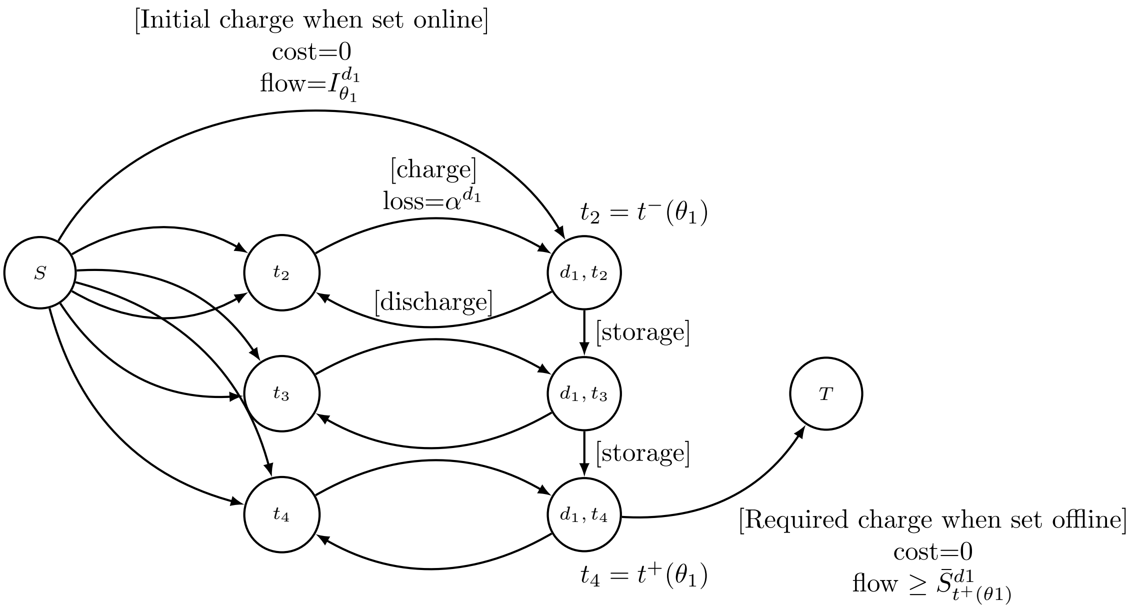

Figure 4: How storage can be modeled in the network.